AMS Foundation Exam (January 2018): Probability Questions

Solve any three of the following four problems.

All problems are weighted equally. On this cover page write which three problems you want graded.

problems to be graded:

Name (PRINT CLEARLY), ID number

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1. Let X and Y be two independent exponential random variables with parameters λ and μ, respectively. Compute the probability $P(X \leq Y | \min\{X, Y\} = x)$.

2. You wrote n letters to each of your n friends and you have an envelope for each of these letters. Suppose the n letters are randomly put into the n envelopes. Let X be the the number of letters that have been put into the correct envelopes. Find the probability $P(X = k)$, where $1 \leq k \leq n$.

3. Let X and Y be two independent exponential random variables with parameters λ and μ respectively. Let $U = \min\{X, Y\}$, $V = \max\{X, Y\}$, and $W = V - U$. Are U and W independent? Justify your answer.

4. Suppose Y is a continuous random variable with probability density function $f(y) = \frac{192}{y^4}$ for $y \geq 4$ (0 otherwise). If the conditional distribution of X given $Y = y$ is a uniform distribution on $[0, y]$. Find the conditional probability density function of Y given $X = x$.